Time-Dependent Multiple Backscattering

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ABSTRACT

Multiple backscattering of pulsed light is formulated for linearly polarized incident radiation. The method is based on the selection of a particular geometry in which the primary and nth order backscattering return to the receiver simultaneously.

It is shown that the returned power from water clouds at a range of 1 km due to orders of scattering higher than the second may be neglected in a collimated pulsed lidar system whose field of view is less than 10^{-2} rad.

1. Introduction

Liou and Schotland (1971) have calculated the secondary backscattered power and depolarization from water clouds to investigate the effect of the higher orders of scattering on the backscattered laser return. They have shown that for a beam width of 10^{-2} rad and an incident wavelength in the visible or near visible, the secondary backscattered power for clouds located about 1 km from the lidar is $\sim 5\%$ of the primary backscattered power for a cloud model with mode radius of $4\,\mu$, and $\sim 8\%$ with a mode radius of $8\,\mu$.

The purpose of the present study is to develop a general theory of time-dependent multiple backscattering, including polarization effects, and to evaluate the power returned from higher orders of backscattering. The approach developed in this paper is based on locating the source of each element of scattered light pulse by choosing a specified geometry. For this specified geometry, the light pulse suffering primary and nth order backscattering returns to the collecting aperture simultaneously.

Computations are performed for third-order backscattering from water clouds located 1 km from the receiver. The results indicate that the returned power from water clouds due to third-order scattering is not significant.

2. Formulation

At an instant of time, in addition to the primary backscattering, there may be a simultaneous contribution at the collecting aperture due to the effect of multiple scattering. In order for nth order scattering to be received at the collecting aperture simultaneously with the primary backscattering, the following time restriction must hold:

$$\frac{2R_0}{c}=\frac{1}{c}(z+R_1+R_2+\cdots+R_{n-2}+R_{n-1}+R_n)=t_n, \quad (1)$$

where $2R_0$ represents the round trip of the primary pulse, z is the distance for first scattering, $R_n (n\geqslant 1)$ denotes the distance from the nth scattering volume to $(n+1)$ scattering volume, c is the velocity of light, and t_n the return time.

Referring to Fig. 1, we consider a particular geometry such that the scattering distances z, R_1, R_2, $\ldots$, R_{n-2} are fixed. Then, on a time-dependent basis as discussed by Liou and Schotland, the corresponding volume in which this entire volume can scatter back to the receiver at a given instant of time, is contained between two confocal ellipsoids with two foci at O and O′. This, of course, is based on the assumption that the volume of the pulse is so small that the variation of the scattering angle may be negligible.

If we keep locating the source of each element of the pulsed light before $(n-1)$ times of scattering, it is easily seen, on a time-dependent basis, that the path of the pulsed light scattered $(n-2)$ times must correspond to a shell of two spheres in which the separation of the shell is the pulse length Δh. A similar argument may be applied for the pulsed light scattered $(n-3)$ times, and so forth.

Having this particular geometry in mind, the flux density per unit volume for each order of scattering, $\mathbf{F}_v^{(n)}$, may be expressed in the general form

$$\mathbf{F}_v^{(n)}=\frac{\beta_s}{R_n^2}\mathbf{P}(\theta_n)\mathbf{L}(\phi_n)\mathbf{F}^{(n-1)}\exp(-\tau_n), \quad n\geqslant 1, \quad (2)$$

where β_s is the volume scattering cross section, R_n the photon path due to nth order of scattering, τ_n the corresponding optical thickness, and $\mathbf{P}(\theta_n)$ and $\mathbf{L}(\phi_n)$ are the phase and rotation matrices (Chandrasekhar, 1960), respectively. The scattering and azimuth angles are θ_n and ϕ_n, respectively, while $\mathbf{F}^{(n-1)}$ represents the flux density due to $(n-1)$th order of scattering.

From (1), $R_n+R_{n-1}=$constant. The flux density scattered n times from a volume of particles, which is found to be formed by two confocal ellipsoids, is (for details, see Liou and Schotland, 1971)

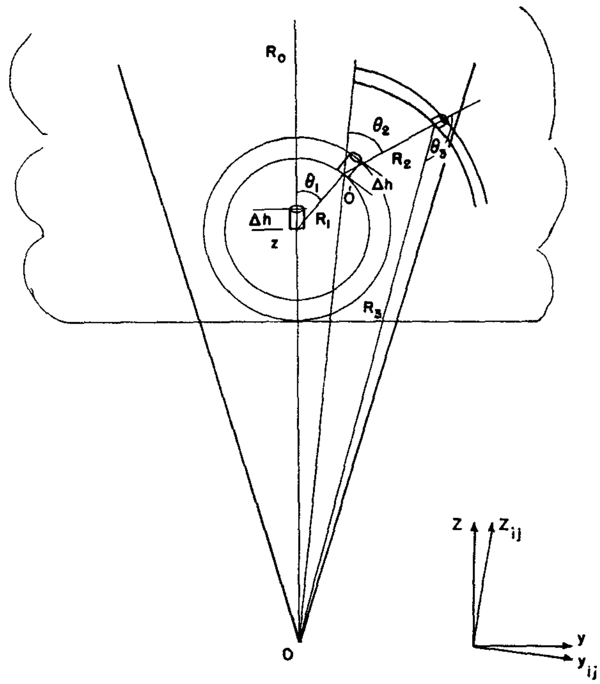

FIG. 1. The physical geometry of the multiple (third-order) backscattered radiation in a collimated pulsed light system. All symbols are explained in the text.

$$\mathbf{F}^{(n)}=\sum_i\sum_j \Delta v_{ij}\beta_s^n\int_{V_{ij}}\frac{1}{R_n^2R_{n-1}^2}\mathbf{P}(\theta_n)\mathbf{P}(\theta_{n-1})$$
$$\times\mathbf{L}(\phi_{n-1})\mathbf{F}^{(n-2)}\exp[-(\tau_n+\tau_{n-1})]dV, \quad (3)$$

where the double summation sums up all possible energy scattered by each ij sub-pulse, the sub-pulses being obtained by dividing the outgoing pulses into several small elements from geometry, with i and j denoting the number of the sub-pulses in the vertical and horizontal directions, respectively. Other symbols are as follows: Δv_{ij} is an element volume of the sub-pulse, V_{ij} represents the integration over the volume bounded by two confocal ellipsoids, and $\mathbf{F}^{(n-2)}$ is the flux density scattered $(n-2)$ times on a time dependent basis. This latter term can be written as

$$\mathbf{F}^{(n-2)}=\int_{V_{n-2}}\frac{\beta_s}{R^2_{n-2}}\mathbf{P}(\theta_{n-2})$$
$$\times\mathbf{L}(\phi_{n-2})\mathbf{F}^{(n-3)}\exp(-\tau_{n-2})dV_{n-2}. \quad (4)$$

As mentioned earlier, a fixed R_{n-2} corresponds to a complete revolution of a volume between two spheres, and the separation of the two spheres is the length of the pulse Δh in which the top of the pulse and the base of the pulse will return to the receiver at a given instant of time. Thus, the differential volume is

$$dV_{n-2}=R^2_{n-2}\sin\theta_{n-2}dR_{n-2}d\theta_{n-2}d\phi_{n-2}. \quad (5)$$

Since the flux density $\mathbf{F}^{(n-3)}$ for the preceding order of scattering is independent of the rotation angle ϕ_{n-2}, the integration over ϕ_{n-2} may be taken first so that a simple form of the flux density scattered $(n-2)$ times may be derived. For a linearly polarized incident beam after integrating over azimuth angle ϕ_{n-2}, (4) can be expressed as

$$\mathbf{F}^{(n-2)}=\pi\beta_s\int_{\mu_{n-2}}\int_{R_{n-2}}\mathbf{M}(\theta_{n-2})\times F^{(n-3)}\exp(-\tau_{n-2})dR_{n-2}d\mu_{n-2}, \quad (6)$$

where

$$\mathbf{M}(\theta_{n-2})=\begin{bmatrix}P_2(\theta_{n-2})\\P_1(\theta_{n-2})\\0\\0\end{bmatrix},$$

$$F^{(n-3)}=F_l^{(n-3)}+F_r^{(n-3)},$$

$$\mu_{n-2}=\cos\theta_{n-2}.$$

Eq. (6) is a general expression for a linearly polarized incident beam and it can be applied for orders of scattering less than n and $(n-1)$. Physically, the result shows that there is no phase difference for two orthogonal electric vectors so that the state of polarization is always linear. This is due to the ϕ integration over a complete revolution.

If we repeatedly substitute (6) for $n\geqslant 3$ into (3), and note $dV=yd\phi dydz$, we obtain the expression

$$\mathbf{F}^{(n)}=\sum_i\sum_j\Delta v_{ij}\beta_s{}^n\pi^{n-1}\times\int_y\int_z\int_{\mu_{n-2}}\int_{R_{n-2}}\cdots\int_{\mu_1}\int_{R_1}\frac{1}{R_n{}^2R_{n-1}{}^2}\mathbf{N}(\theta_n,\theta_{n-1})\times P(\theta_{n-2})\cdots P(\theta_1)F^{(0)}\exp[-(\tau_1+\cdots+\tau_{n-1}+\tau_n)]\times dR_1d\mu_1\cdots dR_{n-2}d\mu_{n-2}ydzdy, \quad (7)$$

where

$$\mathbf{N}=\begin{bmatrix}P_2(\theta_n)P_2(\theta_{n-1})\\P_1(\theta_n)P_1(\theta_{n-1})\\0\\0\end{bmatrix},$$

$$P(\theta_{n-2})=P_1(\theta_{n-2})+P_2(\theta_{n-2}),\quad n>2,$$

and the integration boundary is

$$y=y(\mu_1,R_1;\cdots;\mu_{n-2},R_{n-2}),$$
$$z=z(\mu_1,R_1;\cdots;\mu_{n-2},R_{n-2}),$$

where y and z can be evaluated from geometric relationships, and $R_1,R_2,\ldots,R_{n-2}$ are the fixed paths mentioned previously.

3. Some computations and discussions

In (6) we let $n=3$. After integrating over the area of the collecting aperature and making some rearrangements, the third-order power transfer function can be obtained. An approximate calculation has been carried out. The purpose of this calculation is to investigate the returned power for orders of scattering higher than the second for a vertically polarized incident beam. Assuming a wavelength of $0.6943\,\mu$, a cloud height of 1000 m, a beam width of 10^{-2} rad, and a particle number density of 100 cm^{-3}, the computations were made for two cloud models (Deirmendjian, 1964) with mode radii of 4 and 8 μ. Fig. 2 represents the ratio of the approximate third-order transfer function $T^{(3)}$ to the second-order transfer function $T^{(2)}$ as a function of return time in seconds. At a given instant of time, the third-order backscattering is received at the collecting aperture simultaneously with the primary and secondary backscattering. As shown in this figure, the probability of the pulsed light being scattered three times and escaping from the cloud is higher for the cloud model with mode radius at 8 μ near the cloud base, but decreases rapidly as the light beam penetrates deeply into the cloud. The effect of attenuation for third-order backscattering is much stronger than that of the secondary backscattering. On the other hand, the attenuation effect of the second- and third-order backscattered radiation for cloud model C4 seems to be quite similar so that the ratio approaches an asymptotic value. In general, in our calculations, we found the value of the third-order power transfer function to be about one order of magnitude less than that of the second-order power transfer function, if the range is $\sim$1 km and the concentration is about a few hundred per cubic centimeter. Since the second-order backscattering is only about a few percent

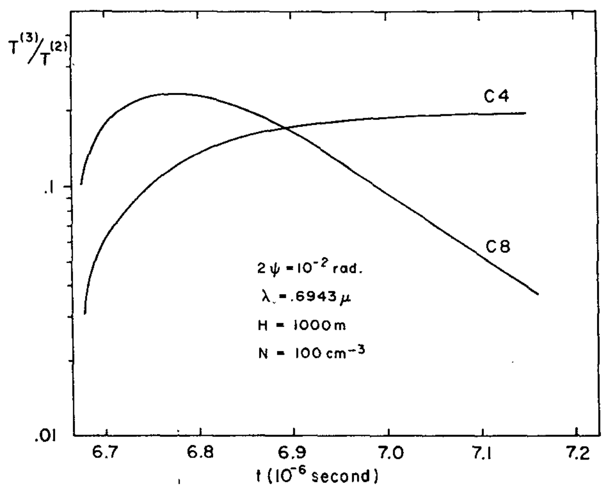

FIG. 2. The ratio of third- to second-order power transfer function, as a function of return time, for two cloud models with mode radii at 4 and 8 μ. Other parameters are indicated in the figure.

of the primary backscattering, it follows that the third-order term is, for all practical purposes, negligible.

Since the same arguments and computing procedures can be applied for the light pulse scattered more than three times, the returned power and depolarization are therefore negligible for orders of scattering higher than the second. Hence, the returned power and depolarization from secondary backscattering can be considered as an "estimation" of the total returned power and depolarization caused by high orders of scattering from spherically symmetrical and uniformly distributed water drops in a collimated pulsed lidar system.

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